St. Jean de Brebeuf Mathematics

CHAPTER 1.3

THE SINE LAW

CHAPTER 1.3 THE SINE LAW

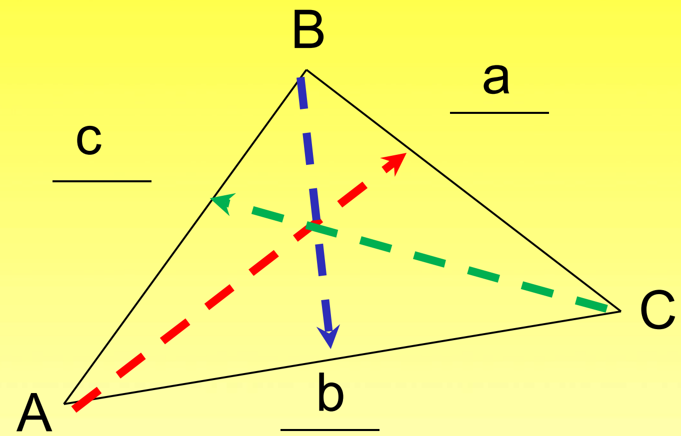
KEY CONCEPTS

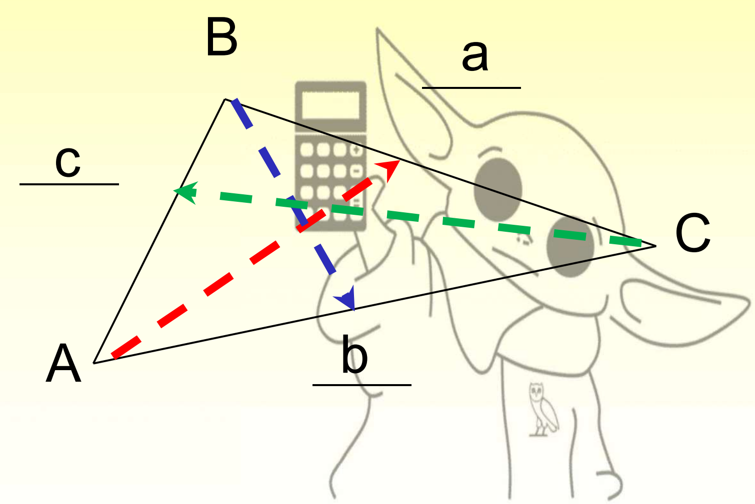
An *acute* or *obtuse* triangle, ABC, can be solved using the **Sine Law** if you know:

→two angle measures and one side measure, where the side measure is *opposite* to one of the given angles

→an angle measure and two side measures, provided one of the sides is *opposite* the given angle

The measure of a side or angle of a triangle can be calculated using a proportion made of two of the ratios from the **Sine Law**





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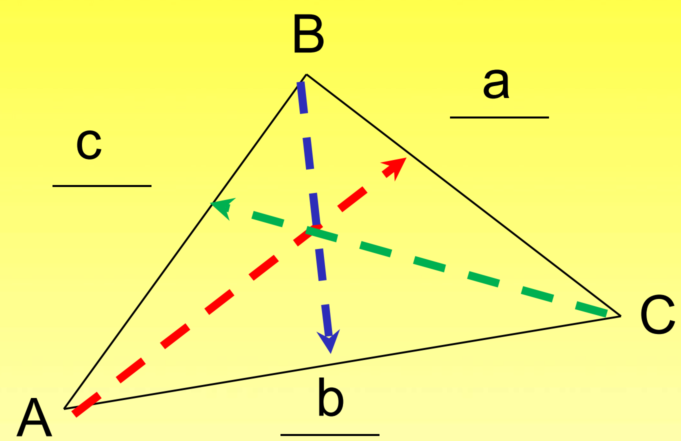
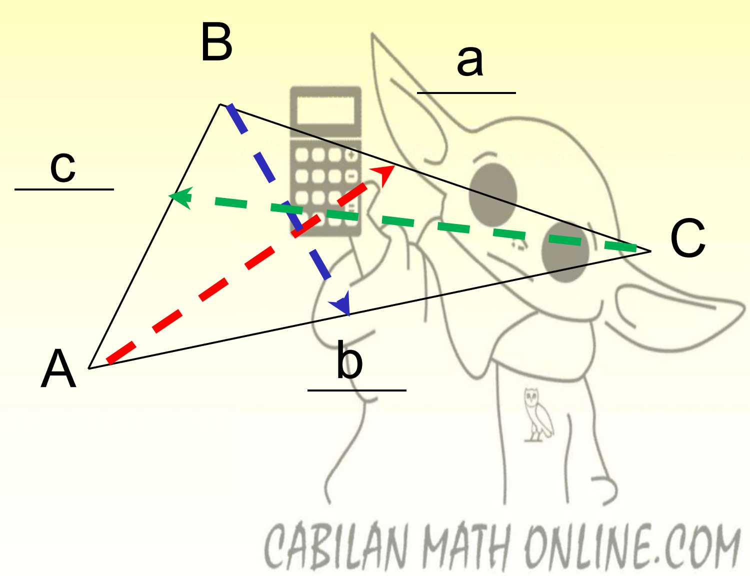
KEY CONCEPTS

FORMULA

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



(by convention, the *lowercase* letters indicate **side length** and the *uppercase* letters indicate **angle measure**)



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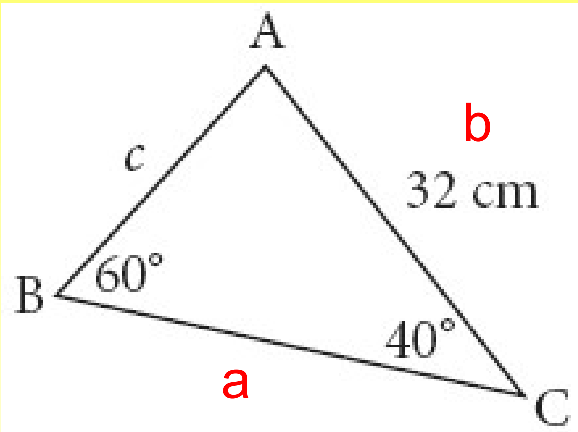
EXAMPLE 1 *Find the Measure of a Side*

Find the measure of side **c** to the nearest centimetre



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{\sin A} = \frac{32}{\sin 60^\circ} = \frac{c}{\sin 40^\circ}$$

$$\frac{32}{\sin 60^\circ} = \frac{c}{\sin 40^\circ}$$

$$\frac{c\sin 60^\circ}{\sin 60^\circ} = \frac{32\sin 40^\circ}{\sin 60^\circ}$$

$$c = 24\,cm$$

BASIC STEPS

1. Write down the formula
2. Substitute known information
3. Set-up a proportion
4. Cross-multiply
5. Solve

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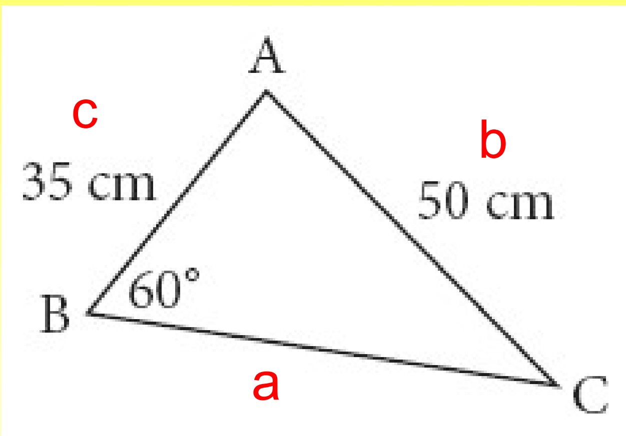
EXAMPLE 2 *Find the Measure of an Angle*

Find the measure of ∠**C** to the nearest degree



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{\sin A} = \boxed{\frac{50}{\sin 60°}} = \boxed{\frac{35}{\sin C}}$$

$$\frac{50}{\sin 60°} = \frac{35}{\sin C}$$

$$\frac{\cancel{50}\sin C}{\cancel{50}} = \frac{35\sin 60°}{50}$$

$$\sin C = \frac{35\sin 60°}{50}$$

$$\sin C = 0.6062$$

$$\angle C = \sin^{-1}(0.6062)$$

Inverse sin
→ 2^{nd}/Shift
then **sin**

$$\boxed{\angle C = 37°}$$

BASIC STEPS

1. Write down the formula
2. Substitute known information
3. Set-up a proportion
4. Cross-multiply
5. Solve

CHAPTER 1.3 THE SINE LAW

EXAMPLE 3

Solving Triangles

Recall that **solving triangles** involves solving for the *unknown* **angles** and **side lengths**

→If <u>two angles</u> are given, solve for the third missing angle (180 – the sum of the given angles)

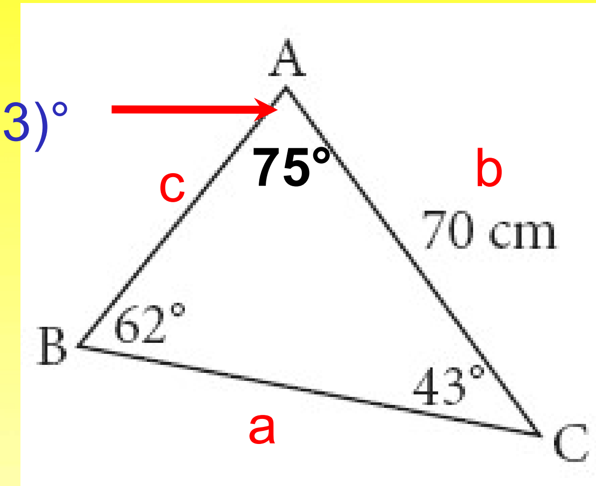
→If <u>one angle</u> and <u>one side</u> are given, use **trigonometric ratios/laws** to determine a side length

Solve the following triangles:

(A)

$180° - (62 + 43)°$

$= \mathbf{75°}$



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EXAMPLE 3
Solving Triangles

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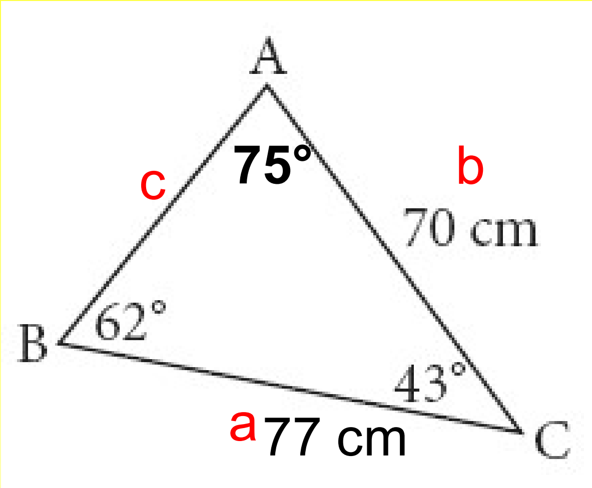
(A)



*Solve for **"a"** first*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 75°} = \frac{70}{\sin 62°} = \frac{c}{\sin 43°}$$

$$\frac{a}{\sin 75°} = \frac{70}{\sin 62°}$$

$$\frac{a \sin 62°}{\sin 62°} = \frac{70 \sin 75°}{\sin 62°}$$

$$a = \frac{70 \sin 75°}{\sin 62°}$$

$$a \doteq 77\ cm$$

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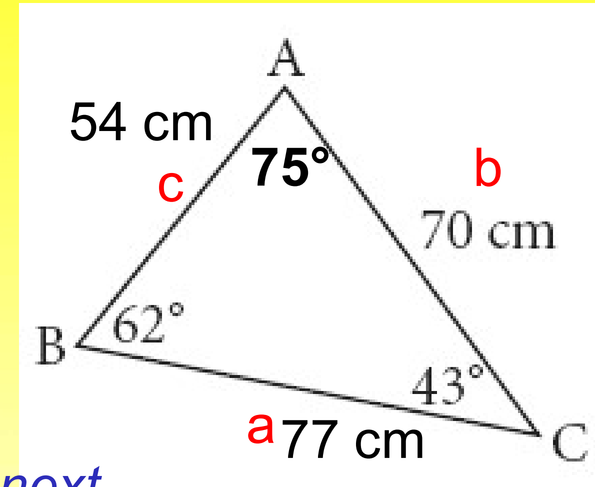
EXAMPLE 3
Solving Triangles

(A)



Recall that **solving triangles** involves solving for the *unknown* **angles** and **side lengths**

→If two angles are given, solve for the third missing angle (180 – the sum of the given angles)

→If one angle and one side are given, use **trigonometric ratios/laws** to determine a side length

Solve the following triangles:

*Solve for **"c"** next*

*→ But use **given info for more accuracy***

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

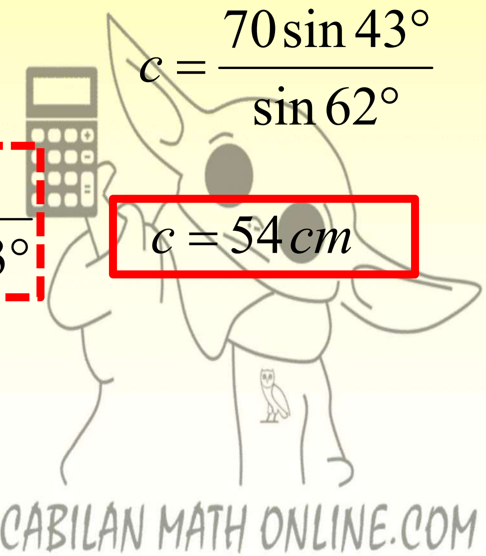
$$\frac{77}{\sin 75°} = \frac{70}{\sin 62°} = \frac{c}{\sin 43°}$$

$$\frac{70}{\sin 62°} = \frac{c}{\sin 43°}$$

$$\frac{c \sin 62°}{\sin 62°} = \frac{70 \sin 43°}{\sin 62°}$$

$$c = \frac{70 \sin 43°}{\sin 62°}$$

$$c = 54\,cm$$



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EXAMPLE 3
Solving Triangles

Recall that **solving triangles** involves solving for the *unknown* **angles** and **side lengths**

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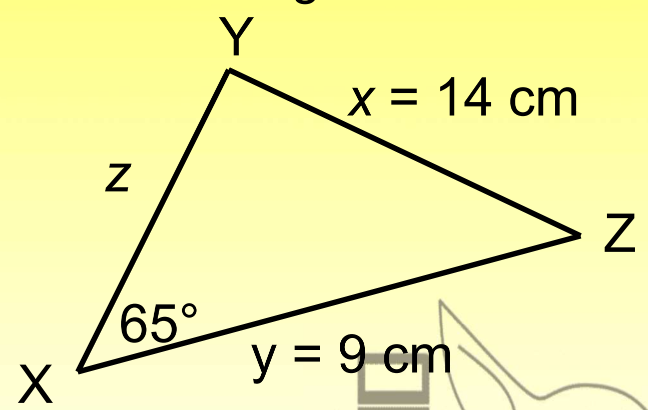
Solve the following triangles:

(B) For a triangle XYZ, **∠X = 65°**, ***x* = 14 cm**, and ***y* = 9 cm**. Need to be opposite to each other!

Using the information above, draw △**XYZ** and solve the triangle



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EXAMPLE 3
Solving Triangles

Recall that **solving triangles** involves solving for the *unknown* **angles** and **side lengths**

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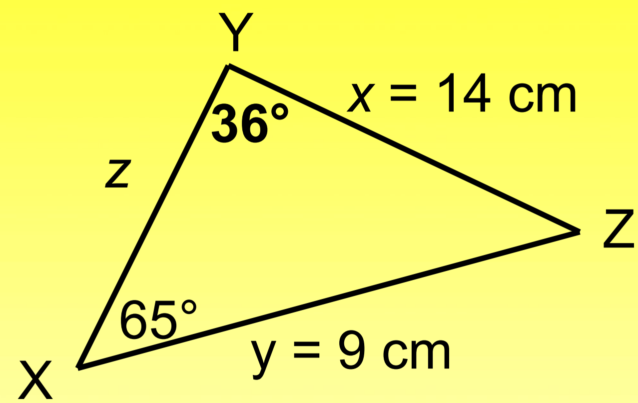

$$\frac{x}{\sin X} = \frac{y}{\sin Y} = \frac{z}{\sin Z}$$

$$\frac{14}{\sin 65°} = \frac{9}{\sin Y} = \frac{z}{\sin Z}$$

*Need to solve for ∠**Y** first!*

$$\frac{14}{\sin 65°} = \frac{9}{\sin Y}$$

$$\frac{14 \sin Y}{14} = \frac{9 \sin 65°}{14}$$

$$\sin Y = \frac{9 \sin 65°}{14}$$

$$\sin Y = 0.5826$$

$$\angle Y = \sin^{-1}(0.5826)$$

$$\angle Y = 36°$$

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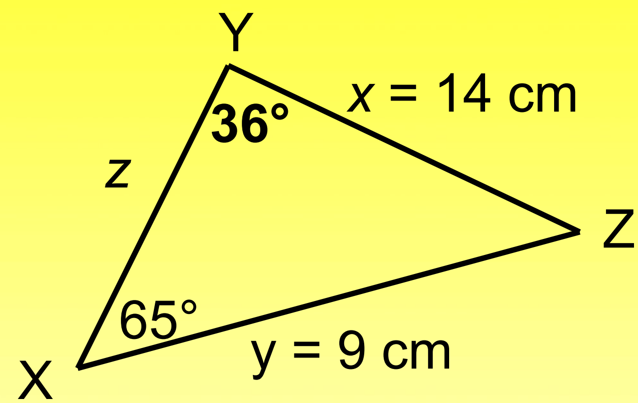

$$\frac{x}{\sin X} = \frac{y}{\sin Y} = \frac{z}{\sin Z}$$

$$\frac{14}{\sin 65°} = \frac{9}{\sin 36°} = \frac{z}{\sin Z}$$

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EXAMPLE 3

Solving Triangles

Recall that **solving triangles** involves solving for the *unknown* **angles** and **side lengths**

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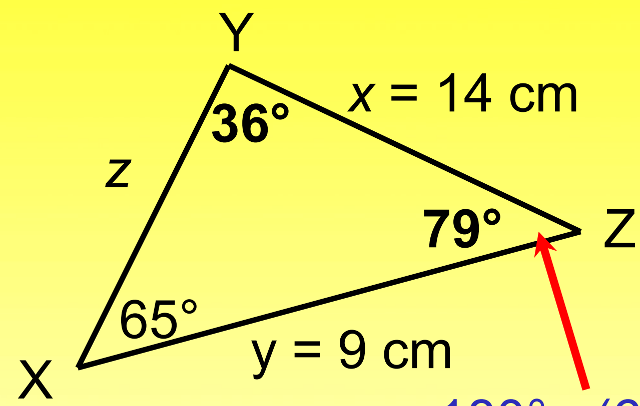
Solve the following triangles:



180° - (65 + 36)°
= **79°**

$$\frac{x}{\sin X} = \frac{y}{\sin Y} = \frac{z}{\sin Z}$$

$$\frac{14}{\sin 65°} = \frac{9}{\sin 36°} = \frac{z}{\sin Z}$$

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EXAMPLE 3

Solving Triangles

Recall that **solving triangles** involves solving for the *unknown* **angles** and **side lengths**

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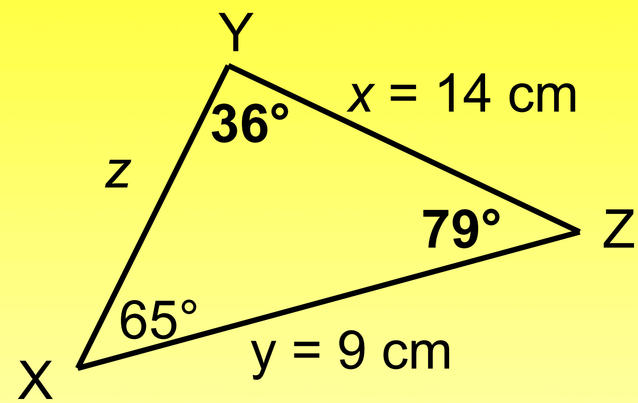

$$\frac{x}{\sin X} = \frac{y}{\sin Y} = \frac{z}{\sin Z}$$

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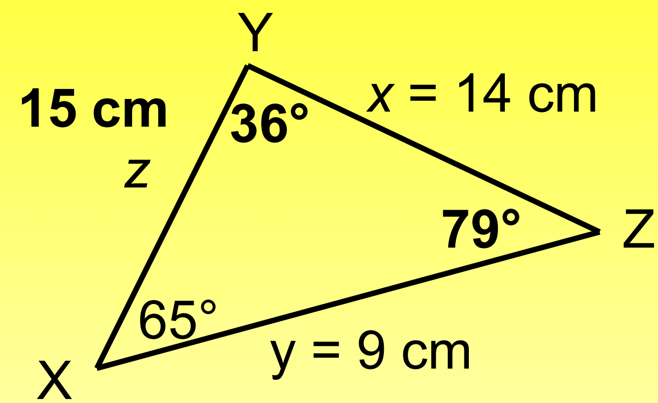
EXAMPLE 3
Solving Triangles



Recall that **solving triangles** involves solving for the *unknown* **angles** and **side lengths**

→If <u>two angles</u> are given, solve for the third missing angle (180 – the sum of the given angles)

→If <u>one angle</u> and <u>one side</u> are given, use **trigonometric ratios/laws** to determine a side length

Solve the following triangles:

$$\frac{x}{\sin X} = \frac{y}{\sin Y} = \frac{z}{\sin Z}$$

$$\frac{14}{\sin 65^\circ} = \frac{9}{\sin 36^\circ} = \frac{z}{\sin 79^\circ}$$

*Solve for "**z**" but use original info for accuracy!*

$$\frac{14}{\sin 65^\circ} = \frac{z}{\sin 79^\circ}$$

$$\frac{z \sin 65^\circ}{\sin 65^\circ} = \frac{14 \sin 79^\circ}{\sin 65^\circ}$$

$$z = \frac{14 \sin 79^\circ}{\sin 65^\circ}$$

$$z = 15\,cm$$

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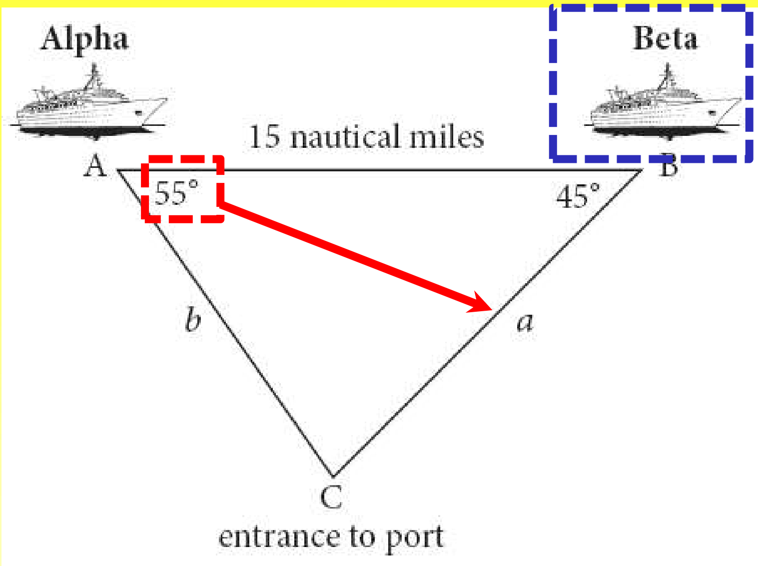
EXAMPLE 4
Solving Problems Using The Sine Law



Two ships are located **15 nautical miles apart**. *Alpha*'s angle to the entrance of the port is **55°** with respect to *Beta*. *Beta*'s angle to the entrance to the port is **45°** with respect to *Alpha*.

(i) Which ship is further to the port entrance? How do you know?

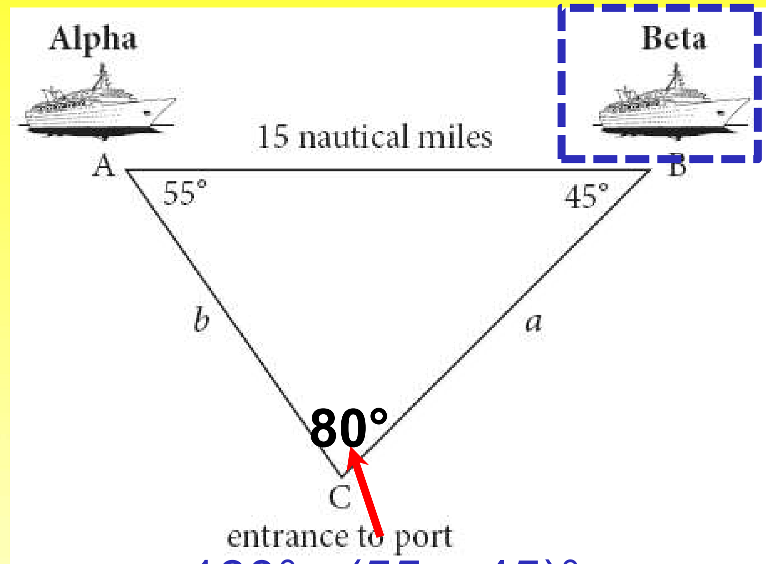
The side *opposite* to the largest angle will be the **longer side**
→**Beta (Ship B)** will be *further* from the port!

EXAMPLE 4

Solving Problems Using The Sine Law

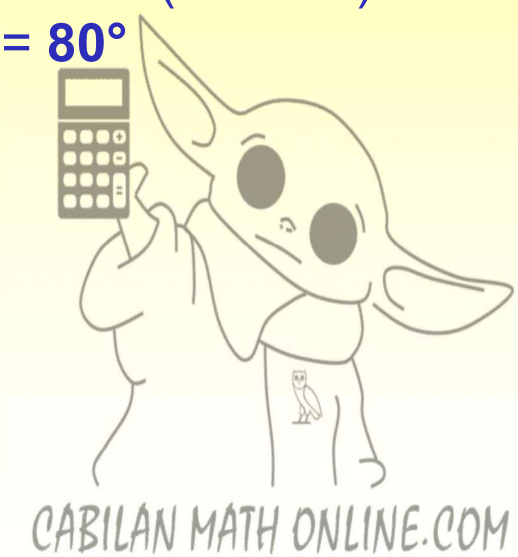
(ii) How far is the ship from Part (i) from port? Round your answer to <u>one decimal place</u>.

→In order to have complete info, we need to solve for ∠C first!!!



$180° - (55 + 45)°$

$= \mathbf{80°}$



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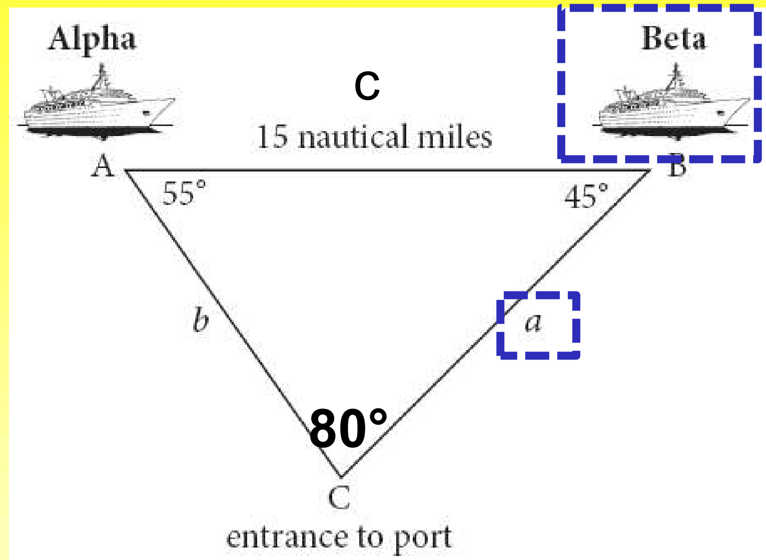
EXAMPLE 4

Solving Problems Using The Sine Law

(ii) How far is the ship from Part (i) from port? Round your answer to <u>one decimal place</u>.



*We will be solving for side **"a"***

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 55°} = \frac{b}{\sin 45°} = \frac{15}{\sin 80°}$$

$$\frac{a}{\sin 55°} = \frac{15}{\sin 80°}$$

$$\frac{a \sin 80°}{\sin 80°} = \frac{15 \sin 55°}{\sin 80°}$$

$$a = \frac{15 \sin 55°}{\sin 80°}$$

$$a = 12.5$$

Ship B will be **12.5 nautical miles** away from the port.

CHAPTER 1.3 **THE SINE LAW**

Homework

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