

ST. JEAN DE BREBEUF MATHEMATICS

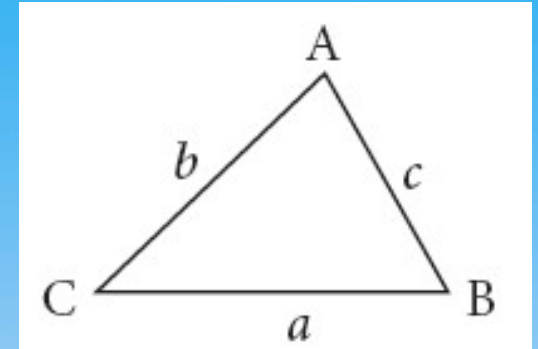
CHAPTER 1.4

THE COSINE LAW

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KEY CONCEPTS

For any triangle, $\triangle ABC$, the **Cosine Law** can be used to solve **side lengths** and **angles**



To find the measure of **any side**, given two sides and the contained angle, the *Cosine Law* can be written as follows (**side-angle-side** or **SAS**)

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

The *Cosine Law* can also be used to find the measure of **an unknown angle**, given three sides (**side-side-side** or **SSS**)

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

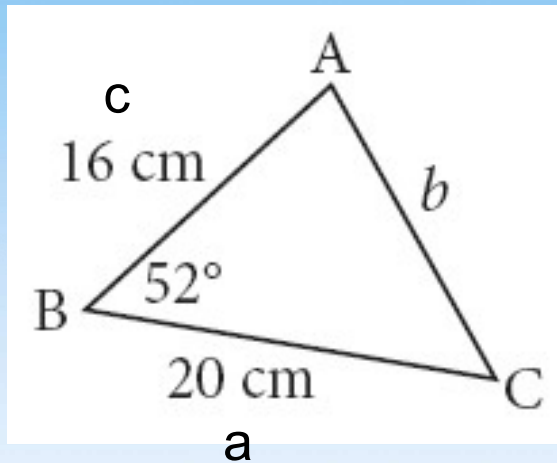
$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

EXAMPLE 1

Finding The Measure of a Side, Given Two Sides and a Contained Angle

Find the measure of the unknown side. Express your answer to one decimal place.



This is **side-angle-side (SAS)**

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$b^2 = (20)^2 + (16)^2 - 2(20)(16)(\cos 52^\circ)$$

$$b^2 = 261.9767$$

$$\sqrt{b^2} = \sqrt{261.9767}$$

$$b = 16.2 \text{ cm}$$



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

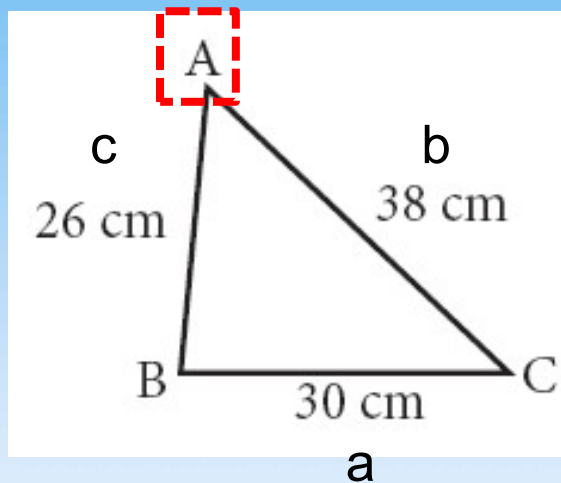
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

EXAMPLE 2 Find the Measure of an Angle, Three Side Lengths Given

Find the measure of $\angle A$ to the nearest degree.



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{(38)^2 + (26)^2 - (30)^2}{2(38)(26)}$$

$$\cos A = \frac{1444 + 676 - 900}{1976}$$

$$\cos A = \frac{1220}{1976}$$

$$\cos A = 0.6174$$

$$\angle A = \cos^{-1}(0.6174)$$

$$\angle A = 52^\circ$$

This is **side-side-side (SSS)**

Inverse cos
 \rightarrow **2nd/Shift then cos**



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$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

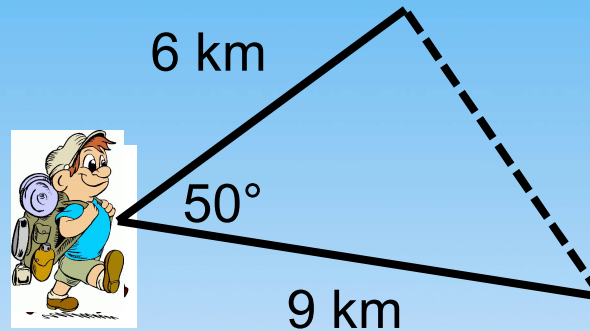
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

EXAMPLE 3

Solving Problems Using the Cosine Law

Two hikers set out in different directions from a marked tree on the Bruce Trail. The angle formed between their paths measures 50° . After 2 h, one hiker is **6 km** from the starting point and the other is **9 km** from the starting point. How far apart are the hikers? Express your answer to one decimal place.

Draw a diagram and solve for the unknown distance



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

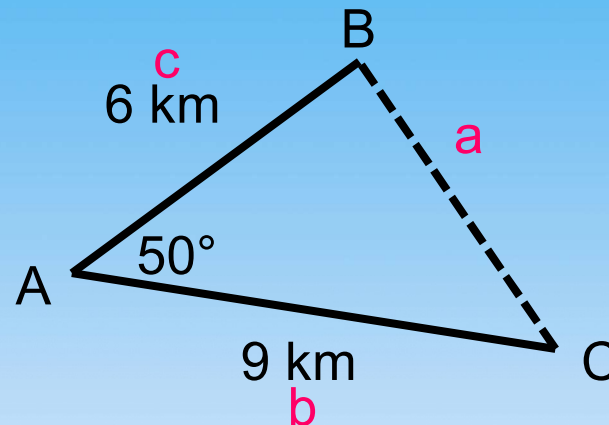
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

EXAMPLE 3

Solving Problems Using the Cosine Law

Two hikers set out in different directions from a marked tree on the Bruce Trail. The angle formed between their paths measures 50° . After 2 h, one hiker is **6 km** from the starting point and the other is **9 km** from the starting point. How far apart are the hikers? Express your answer to one decimal place.

Draw a diagram and solve for the unknown distance



We have **side-angle-side (SAS)**

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = (9)^2 + (6)^2 - 2(9)(6)(\cos 50^\circ)$$

$$a^2 = 47.5789$$

$$\sqrt{a^2} = \sqrt{47.5789}$$

$$a = 6.9$$

The hikers are **6.9 kilometres** apart.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

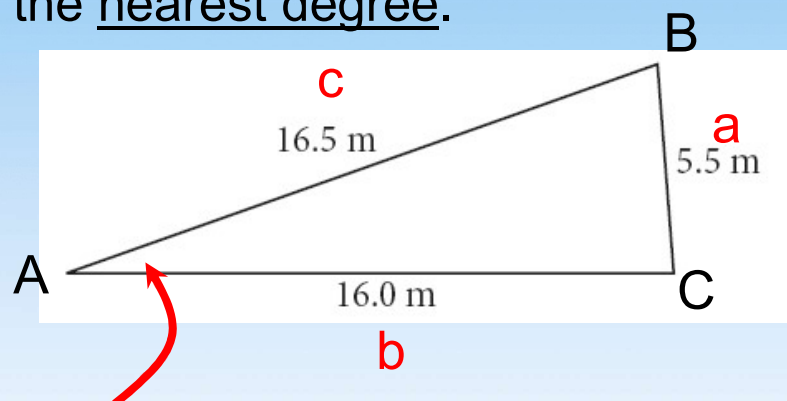
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

EXAMPLE 4 Solving Problems using the Cosine Law
(Page 40, #7)

A motocross ramp is to be built for an upcoming race. The measures for the sides of the ramp are as shown. Calculate the *angle of inclination* of the ramp to the nearest degree.



We are solving for this angle

The angle of inclination is **19°**.

We have **side-side-side (SSS)**

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{(16.0)^2 + (16.5)^2 - (5.5)^2}{2(16.0)(16.5)}$$

$$\cos A = \frac{256 + 272.25 - 30.25}{528}$$

$$\cos A = \frac{498}{528}$$

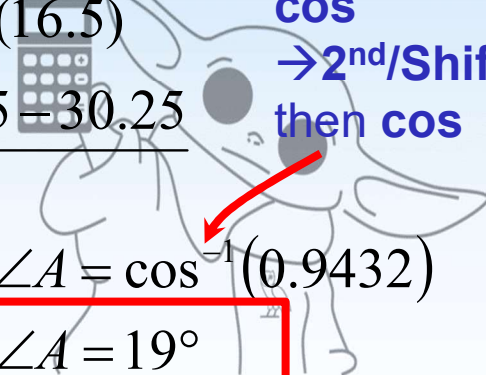
$$\cos A = 0.9432$$

$$\angle A = \cos^{-1}(0.9432)$$

$$\angle A = 19^\circ$$

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Inverse
cos
→ 2nd/Shift
then cos



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Homework

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2ac, 3 – 6, 8 – 10, 12a

