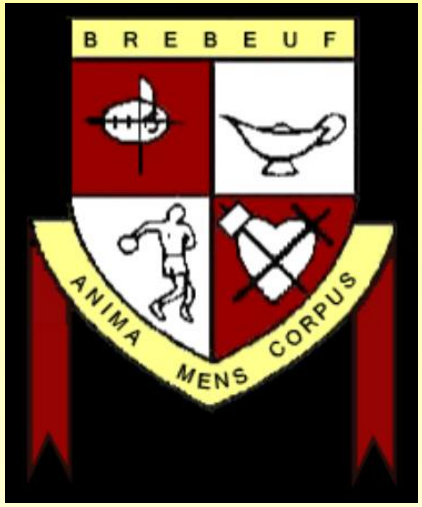
ST. JEAN DE BREBEUF MATHEMATICS



CHAPTER 1.5

MAKE DECISIONS USING TRIGONOMETRY

CHAPTER 1.5 MAKE DECISIONS USING TRIGONOMETRY

KEY CONCEPTS

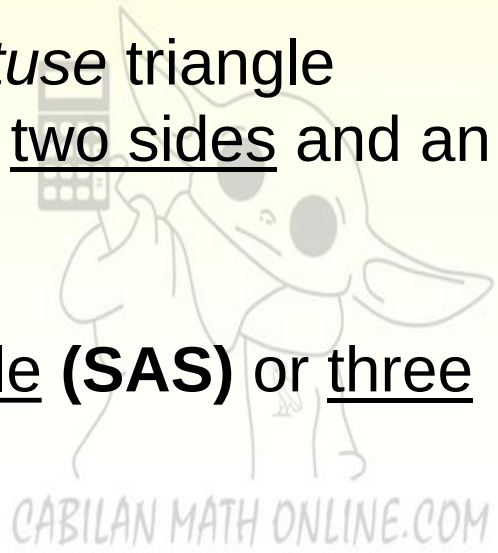
Decide which formula or tool to use based on the type of triangle the situation presents.

If the problem is modelled by a right triangle, use the **primary trigonometric ratios**.

If the problem is modelled by an *acute* or *obtuse* triangle

→with two angles and a given side or two sides and an opposite angle, use the **Sine Law**

→with two sides and a contained angle **(SAS)** or three sides **(SSS)**, use the **Cosine Law**



CHAPTER 1.5 MAKE DECISIONS USING TRIGONOMETRY

BASIC TRIGONOMETRIC RATIOS	THE SINE LAW	THE COSINE LAW
$\sin A = \frac{OPPOSITE}{HYPOTENUSE}$ $\cos A = \frac{ADJACENT}{HYPOTENUSE}$ $\tan A = \frac{OPPOSITE}{ADJACENT}$ If the problem is modelled by a right triangle, use the **primary trigonometric ratios**.	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ If the problem is modelled by an *acute* or *obtuse* triangle **(non-right angled)** →with two angles and a given side or →two sides and an opposite angle, use the **Sine Law**	$a^2 = b^2 + c^2 - 2bc\cos A$ $b^2 = a^2 + c^2 - 2ac\cos B$ $c^2 = a^2 + b^2 - 2ab\cos C$ $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$ $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

PYTHAGOREAN THEOREM

$c^2 = a^2 + b^2$

Pythagorean Theorem is *only* used to solve for a missing side in a right angle triangle

CHAPTER 1.5 MAKE DECISIONS USING TRIGONOMETRY

If the problem is modelled by an *acute* or *obtuse* triangle **(non right angle)**
→with <u>two sides</u> and a <u>contained angle</u> **(SAS)** or
→ **Three sides (SSS)**, use the **Cosine Law**

THE COSINE LAW

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

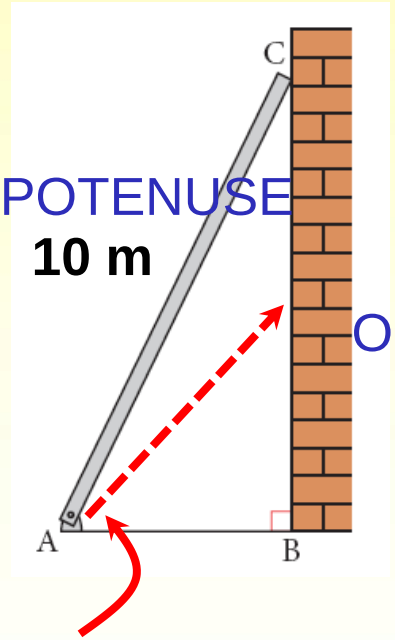
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

CHAPTER 1.5 MAKE DECISIONS USING TRIGONOMETRY

EXAMPLE 1

A **10 m** ladder leans against a wall. The top of the ladder is **9 m** above the ground. What is the *angle of elevation* between the floor and the ladder?



HYPOTENUSE

10 m

9 m

OPPOSITE

Angle of elevation

We have a **RIGHT TRIANGLE**

→Use **basic trigonometric ratios**

→Label the *given* sides (with respect to the angle we are solving for) and determine which trigonometric ratio to use

$$\sin A = \frac{OPPOSITE}{HYPOTENUSE}$$

$$\sin A = \frac{9}{10}$$

$$\sin A = 0.9$$

Inverse sin

→**2nd/Shift** then **sin**

$$\angle A = \sin^{-1}(0.9)$$

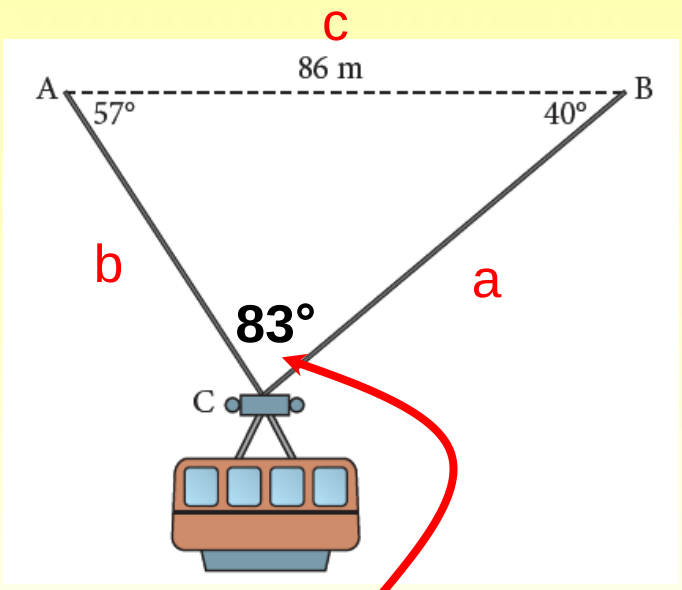
$$\angle A = 64°$$

The *angle of elevation* between the floor and the ladder is **64°**.

CHAPTER 1.5 MAKE DECISIONS USING TRIGONOMETRY

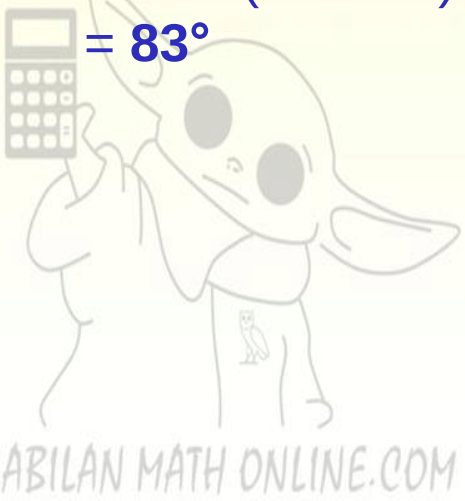
EXAMPLE 2

A cable car stops part of the way across an **86 m** wide gorge. The cable holding the car makes an *angle of depression* of **57°** at one end and an *angle of depression* of **40°** at the other end. How long is the cable that holds the car? Round your answer to the nearest metre.



*Need to solve for **"a"** and **"b"** and add them together!*

180° - (57 + 40)°
= **83°**



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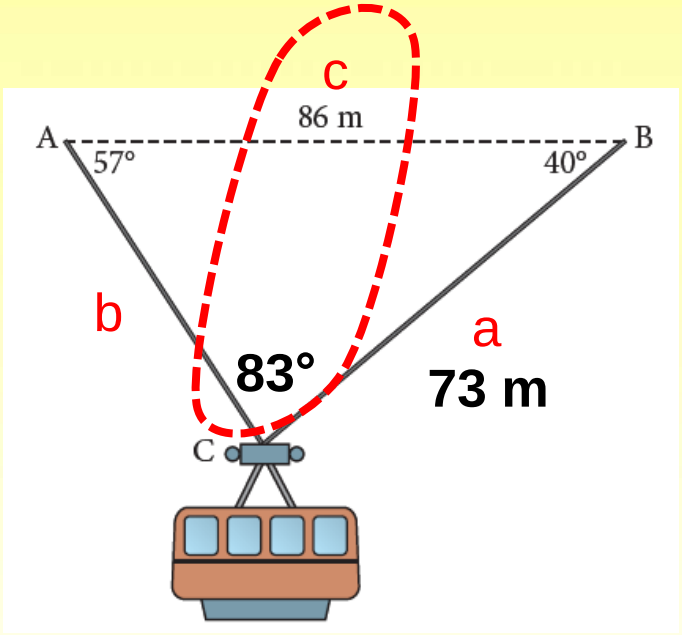

*Need to solve for **"a"** and **"b"** and add them together!*

We have an **angle** and a side *opposite* to the angle
→Use **SINE LAW**!

*For **"a"***

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$\frac{a}{\sin 57^\circ}=\frac{b}{\sin 40^\circ}=\frac{86}{\sin 83^\circ}$$

$$\frac{a}{\sin 57^\circ}=\frac{86}{\sin 83^\circ}$$

$$\frac{a\sin 83^\circ}{\sin 83^\circ}=\frac{86\sin 57^\circ}{\sin 83^\circ}$$

$$a=\frac{86\sin 57^\circ}{\sin 83^\circ}$$

$$a=73\,m$$

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EXAMPLE 2

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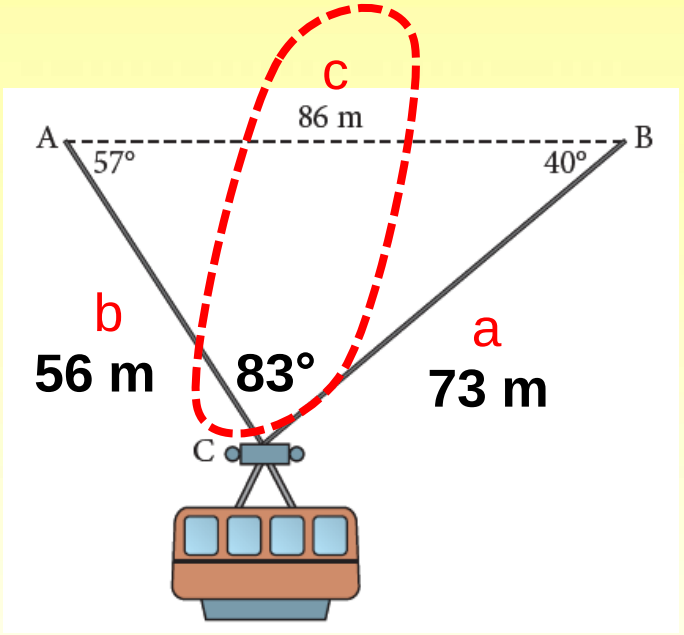


*Need to solve for **"a"** and **"b"** and add them together!*

→Use **SINE LAW**!

*For **"b"** (use given info for accuracy!)*

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$\frac{73}{\sin 57^\circ}=\frac{b}{\sin 40^\circ}=\frac{86}{\sin 83^\circ}$$

$$\frac{b}{\sin 40^\circ}=\frac{86}{\sin 83^\circ}$$

$$b\sin 83^\circ=86\sin 40^\circ$$

$$b=\frac{86\sin 40^\circ}{\sin 83^\circ}$$

$$b=56\,m$$

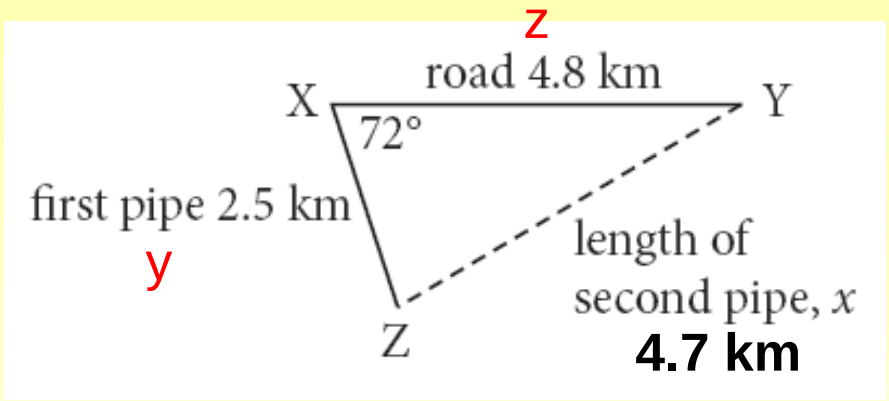
The length of the cable that holds the car is **129 metres**.

For length of cable
= 73 m + 56 m
= 129 m

CHAPTER 1.5 MAKE DECISIONS USING TRIGONOMETRY

EXAMPLE 3

A sewer pipe for a new subdivision has to be laid underground. A connection is made to the main service pipe at either end of the **4.8 km** stretch of road. One pipe, **2.5 km** long, makes an angle of **72°** at one end of the road.



(a) Calculate the length of the second pipe

We have an **acute/non-right triangle**
→**side-angle-side (SAS)**
→ Use **COSINE LAW**!

$$x^2 = y^2 + z^2 - 2yz\cos X$$

$$x^2 = (2.5)^2 + (4.8)^2 - 2(2.5)(4.8)(\cos 72°)$$

$$x^2 = 21.8736$$

$$\sqrt{x^2} = \sqrt{21.8736}$$

$$x = 4.7$$

The length of the second pipe is **4.7 km**

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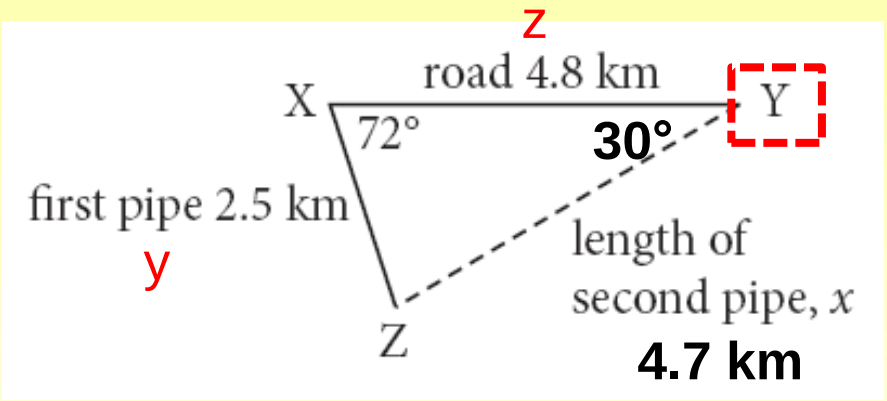
(b) Determine the angle **between the second pipe and the road**

Need to solve for ∠**Y**
→ Easiest to use **SINE LAW**!

The angle between the second pipe and the road is **30°**.



$$\frac{x}{\sin X} = \frac{y}{\sin Y} = \frac{z}{\sin Z}$$

$$\frac{4.7}{\sin 72°} = \frac{2.5}{\sin Y} = \frac{4.8}{\sin Z}$$

$$\sin Y = 0.5059$$

$$\frac{4.7}{\sin 72°} = \frac{2.5}{\sin Y}$$

$$\angle Y = \sin^{-1}(0.5059)$$

$$\angle Y = 30°$$

$$\frac{4.7 \sin Y}{4.7} = \frac{2.5 \sin 72°}{4.7}$$

Inverse sin
→2nd/Shift then **sin**

$$\sin Y = \frac{2.5 \sin 72°}{4.7}$$

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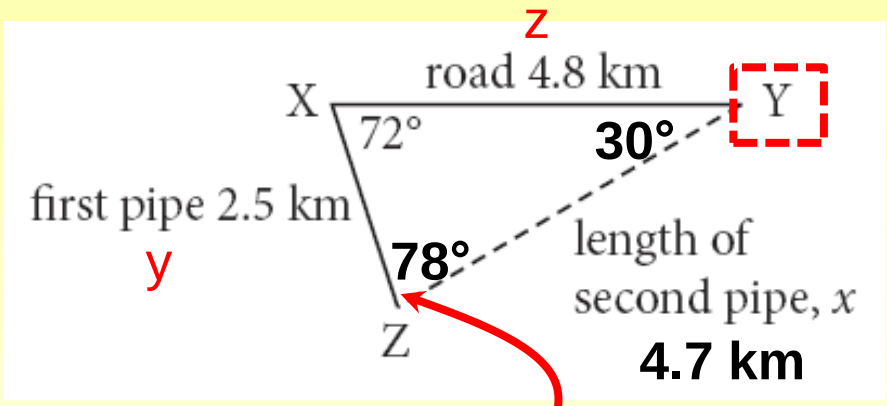
(c) What is the angle **between the first pipe and second pipe**?

The angle between the first and second pipe is **78°**.



180° - (72 + 30)°
= **78°**

Homework

Page 48 – 51 #1 – 4, 6 – 8, 10

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BASIC TRIGONOMETRIC RATIOS

$$\sin A = \frac{OPPOSITE}{HYPOTENUSE} \qquad \cos A = \frac{ADJACENT}{HYPOTENUSE} \qquad \tan A = \frac{OPPOSITE}{ADJACENT}$$

SINE LAW

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

COSINE LAW

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

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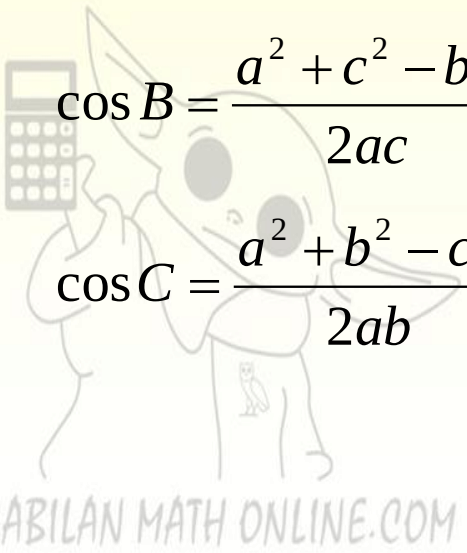

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